A field based representation for vague areas defined by spatial prepositions

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Abstract

Natural language is one of the primary means of communicating spatial information, but existing geographic information retrieval (GIR) facilities are weak in this respect. One of the major challenges in automated interpetation of spatial natural language is how to model the regions implied by spatial expressions. This paper presents a field-based model for representing the vague regions defined by spatial language, including a method for defining field values from existing spatial language data sources. To interface this new vague field model with existing GIR systems and methods, an algorithm for extracting crisp boundaries from the field representation is also presented.

1. Introduction

Natural language is one of the primary means of communicating spatial information, but existing computing facilities for accessing geographically referenced information have limited capability for understanding spatial language except within the context of highly structured user interfaces. The quality of web search tools for accessing geographical information would be much improved if these systems could intelligently process queries that contained spatial prepositions such as near, in front and north when used in combination with named places.

A number of hurdles need to be overcome to succeed in intelligent interpretation of spatial natural language. A major challenge is to model spatial language, especially the spatial relations primarily defined by spatial prepositions. The regions that are referred to by the spatial relations have vague boundaries, and their extent is context dependent, varying with the types of phenomena involved and with the events taking place. Part of the vagueness arises from the fact that for a given situation many factors influence whether a particular spatial relation applies and there are no standard definitions for the use of the different prepositions that may apply. It is also the case that for most spatial prepositions there will be no agreement between users on exactly what is the form and extent of the space referred to. Indeed a single individual may not have a precise view of the region referred to.

This paper presents a field-based model for vague spatial relations and also a method for creating a crisp boundary from the field for integration with existing GIS methods. To avoid errors introduced by mixing knowledge from different domains within language, only spatial relations as used in image captions will be used.

Section 2 provides background information on spatial language and models, section 3 and 4 present the field based model and crisping algorithm and section 5 contains concluding remarks and an outlook to future work.

2. Representing vague areas

When dealing with spatial phenomena, the need to handle vagueness is unavoidable. A number of sources exist for this vagueness

- Multivariate classification vagueness due to multiple non-independant classification criteria;
- Multiperson disagreement vagueness due to different interpretations by people, see (Montello et al., 2003) and (Robinson, 2000);
- Natural vagueness vagueness due to the sorites paradox as illustrated by (Fisher, 2000);
- Precision vagueness due to representational and scale factors.

For a more detailed classification of vagueness in the context of geographical information, see (Evans and Waters, 2007).

2.1. Spatial models

Initial spatial models represented space in a crisp way. (Egenhofer, 1989) used a set-theoretic approach to represent the topological relations between two crisp regions. A second early topological model is the Region Connection Calculus (RCC) defined by (Randell et al., 1992), based on the connectivity relation C(x, y) between two regions. Both models work well for the fiat and bona-fide boundaries as defined by (Smith and Varzi, 1997), but many of the spatial regions people encounter in practice are not crisp.

To deal with this vagueness inherent in human geography a number of extensions to the crisp models and also new vague models were defined. (Cohn and Gotts, 1996) created the *egg-yolk* model which is an extension of the RCC model. The *egg-volk* model introduces a broad border region which defines the area that partially but not fully belongs to the core region. To what degree points within this border region are elements of the core region is not defined, only that they are no longer fully part of the core region and not yet part of the area outside. This provides a notion of vagueness while retaining the simplicity of reasoning with crisp regions. With a similar approach (Clementini and Felice, 1996) extend the 9-intersection model to deal with such broad-boundary regions. The two models differ in what relations they support, but are of similar expressivity. There are other approaches that also lead to a broad-boundary model (Schneider, 1996) (Bennet, 2001) (Kettani and Moulin, 1999) (Kulik, 2001), the attraction of this approach being that they allow the basic modelling of vagueness without the complexity of having to consider the inherent characteristics of the vagueness or its relationship to real-world phenomena.

To model the details of how the vagueness works, fuzzy sets have been proposed as a solution. Fuzzy sets were introduced by (Zadeh, 1965) and are an extension of classical set theory. Instead of only providing a boolean member/nonmember definition of a set, fuzzy sets provide a membership function $\mu : X \to [0,1]$, where 0 is classical notmember and 1 is classical member-of. In GIS (Schneider, 1999) provides a definition for fuzzy points, lines and regions. This definition is extended in (Schneider, 2000) to a full algebra for fuzzy regions. Fuzzy sets are harder to handle than crisp sets, but (Schneider, 1999) shows how fuzzy sets can be reduced to a (possibly infinite) set of α -cuts. An α -cut is a crisping of a fuzzy set at an arbitrary value (the α value), where only those elements with membership values higher than α are part of the cut, thus deriving a classical crisp set from the original fuzzy set. The advantage of this approach is that all the existing work on crisp regions can be applied to such a set of α -cuts.

When using fuzzy models to represent real world phenomena, the hardest problem is the definition of the membership function. (Robinson, 2003) gives an overview of different methods for defining the membership function. The approach taken by most is to use one of the standard membership functions to approximate the actual membership function. The properties of these standard functions are well understood and they are easy to handle. (Schockaert et al., 2008) follow this path in their work on modelling phrases such as "within walking distance". They use a standard trapezoidal membership function to approximate the data they mined from the web. Similarily (Mukerjee et al., 2000) use human input to modify the shape of a standard ellipsoid field representing the extent of a spatial relation.

2.2. Spatial language

Apart from maps and other graphical representations of space, the primary means for exchanging spatial information is natural language. The issues arising from spatial language and its use in accessing geographic information were raised by (Frank and Mark, 1991). The primary elements used in spatial language are count nouns referring to objects and spatial prepositions defining the spatial relations between the objects (Landau and Jackendoff, 1993). In image caption spatial language the role of the objects is taken by toponyms referring to places that are linked via the spatial prepositions. This paper focuses only on representing and handling the vague areas introduced by spatial prepositions and not on the representations of the places referred to by toponyms.

Spatial relations relate at least two objects to each other as in "A pond north of Stackpole". In this paper the object that acts as the reference object, "Stackpole" in this case, will be referred to as the *ground*, while the referred object "pond" will be called the *figure*. In the kind of spatial relations used in image captions, the *figure* object usually describes the content of the image, while the *ground* refers to a toponym in the close vicinity.

(Landau and Jackendoff, 1993) showed that the number of spatial prepositions is very small compared to the number of names for shapes and locations. Thus in order to be able to describe all possible configurations, they must be very flexible with respect to the situations they can be applied to. As (Herskovits, 1985) illustrates that means that it is very hard to cleanly define how and when they can be applied, using only a simple relations based approach.

Various models have been proposed to represent and reason with spatial relations derived from natural language. (Frank, 1996) describes an algebra for reasoning on the cardinal directions. The algebra can deal with a four direction (N, E, S, W) or an eight direction (N, NE, E, SE, S, SW, W, NW) model and allows answering queries of the kind "A is north of B and B is east of C. What is the relation between A and C?". (Goyal and Egenhofer, 2000) provide a similar eight direction model, extending the Frank model by allowing the regions involved in the cardinal relation to cover more than one direction. Other qualitative models for the cardinal directions can be found in (Ligozat, 1998) and in (Kulik et al., 2002) who introduce a ranking method for objects involved in a cardinal direction. (Hernández, 1991) describes a model for qualitatively representing spatial relations in an indoor context.

These models take a purely qualitative view of the spatial relations, and do not address the issue of geometric extent implied by the spatial relations. For modelling the quantitative aspects, fuzzy sets are advantageous as they avoid having to create a crisp approximation of the representation too early in the analysis process (Altman, 1994). (Robinson, 2000) uses a fuzzy approach to model nearness at the inter-town scale. Users are presented with a set of questions "Is town A close to town B?", with both towns shown on a map and the users answer *yes* or *no*. This builds a representation of nearness for one user, which can then be combined for multiple users to produce a general fuzzy representation of nearness.

One issue that is not addressed in this paper is how strongly language and culture influence spatial representation and reasoning. (Mark et al., 2007) and (Levinson, 2003) make a very strong case for the influence of space on language and vice-versa. Contrasting that (Xiao and Liu, 2007) and (Ragni et al., 2007) found that for non-linguistic tasks focusing on latitude estimates and topological classification, no significant differences between different cultures exist. While the methods presented in this paper are applicable to any language, the data presented here is taken from UK English image captions and thus only directly applicable to UK English image caption spatial language.

3. A field-based model for spatial relations

As (Couclelis, 1992) illustrates, geographic space can be seen from a vector or object, or from a raster or field perspective. While currently the object view dominates in GIS software, most of the data handled by these systems are actually more field-like in nature. This is especially true for the spatial relations this paper focuses on, which describe regions in which the membership to the spatial relation varies across the whole region. (Nishida et al., 1987) describe a field model for placing objects in a spatial scene, but how empirical data would work into the field is unclear. For representing spatial relations such as "north of", a field based approach has one strong advantage over traditional crisp and egg-yolk models, in that it can accurately represent the level of membership at each point and does not only provide a rough approximation. The fuzzy models described earlier are much closer to a field representation. but the field model does offer an advantage. The hardest problem for fuzzy models is determining the membership function. Also it is difficult to combine multiple membership functions into one final result, in those cases where multiple factors are relevant to the membership. A field based model avoids both problems, as the field values can be taken directly from the data source, without having to be fit into a functional model first.

The field model for spatial relations is represented as an $n \times n$ matrix, with the ground for the spatial relation usually located at $(\frac{n}{2}, \frac{n}{2})$, although that is not necessary and in the examples presented, the ground is placed further towards the bottom of the field to improve image clarity. Each field cell holds a membership value that is defined on the interval [0, 1], with 0 signifying no membership and 1 complete membership to the spatial relation. Depending on the spatial relation used the size of each raster cell can be varied, but for the spatial relations described here, a cell size of 50x50m has been determined to be the best compromise between spatial resolution and computational complexity.

Using this model a number of spatial relations have been modelled, with the results for "north of" presented here. The raw data that the field is based on, were taken from the Geograph project. The Geograph¹ project aims to cover each square kilometre of the UK with a representative photograph. These photographs contain a caption and location information from GPS units. Since the aim is to create representative photographs, the captions chosen tend to be spatial in nature, describing the location of the photograph and not only its contents. As such the project represents an ideal source of spatial linguistic information. The Geograph project has provided a database dump of roughly 350,000 records, containing image captions and location information, but not the actual images themselves.

The first step in constructing the field is extracting uses of the desired spatial relation from the Geograph captions. $GATE^2$ is used for part of speech tagging and the identification of spatial relations. Identifying toponyms in captions is not an easy task, but based on an analysis of the image captions, a simple metric was devised. Any word that starts with an uppercase letter is assumed to be a candidate toponym. Combined with a list of excluded words such as "A" and "The", this metric provides good results.

The tagged captions are then matched against patterns of the form "<spatial relation> <toponym>", in this case "north of <toponym>". The hypothesis employed was that the GPS coordinates of the image and the location of the toponym matched by the pattern formed one valid use for the spatial relation. As each spatial relation appears multiple



Figure 1: Field model representation: Initial point cloud, smoothed field (\times marks the ground toponym location)

times it is possible to build up a set of valid uses, which then feed into the field representation.

The toponyms are geocoded using the Geonames.org service, which returns a point representation of the centre of the toponym location. No toponym disambiguation was performed, except for only accepting exact toponym matches. As the distances involved in the spatial language of image captions tend to be short (most less than 5km), an incorrect disambiguation is immediately clear as a statistical outlier. For each of the patterns the GPS co-ordinates of the image and the location of the toponym are combined to calculate the angle and distance of the image location from the ground toponym.

These distance/angle pairs are then plotted onto the field, relative to the ground toponym. As this method combines distance and angle data from multiple captions, it is necessary to guarantee that the scale involved in all captions is the same. The area "north of" a point of interest such as a church will have a different scale to that "north of" a town or village. The Geonames.org service in addition to the location of the toponym also provides information on the toponym's type and in the data presented here only toponyms of the type populated place were used. In combination with the fact that when locating images only very local information is used, this gives a high confidence in the data. Figure 1 shows the plot of the spatial relation "north of <toponym>". The plotted field is then smoothed using a 30x30 cell rectangular kernel and the resulting values normalised to the [0, 1] range, so that the crisping algorithm can be applied.

4. Crisping the field model

The field model provides a very powerful representation for spatial relations, but for using the results in other GI systems or applying existing crisp methods, a vector based representation is required. This crisping makes it possible to use a vague representation of "north of" as the input into a crisp spatial query in current GI systems. The crispings should always have meta-data associated with them, that document that the crisp representation is just one possible crisping, and not a normative result for the spatial relation. In fuzzy models α -cuts (Klir and Yuan, 1995) or centre of area methods (Power et al., 2001) (Palanciogla and Beard, 2001) are employed to crisp the fuzzy representation. While the α -cut method could also be applicable to the field model, this paper presents an active contour based crisping algorithm. The advantage of an active contour crisping algorithm is that further constraints and influences

¹http://www.geograph.org.uk

²http://gate.ac.uk

can easily be integrated into the crisping algorithm. Examples of such constraints might be hard boundaries such as shorelines or mountains, or softer influences like the influence of road-connectivity on the shape of the relation, or the conflicting influence of other spatial relations that could be used to describe the location.

Active contour models were first introduced by (Kass et al., 1988) for finding boundaries in image data. They are defined as energy minimising functions, consisting of an internal energy that is responsible for maintaining the active contour's shape and an external energy that represents the data to be modelled (equation 1). In image processing the internal energy is usually defined so as to maintain an even spacing between the control points and also to smoothen the angles at each point (Lam and Yan, 1994). The external energy is then defined by the image being processed. Frequently the energy source is the gradient of the image, the active contour is then attracted to boundaries in the image where the gradient is steepest (Lam and Yan, 1994). These kinds of active contours are frequently employed in medical image feature extraction (Shang et al., 2008).

$$E(s) = E_{int}(s) + E_{ext}(s) \tag{1}$$

Active contours have also been used in the GIS field, (Burghardt, 2005) and (Steiniger and Meier, 2004) use them for line smoothing in map generalisation applications. For the line smoothing used in map generalisation, no external energy is needed. The active contour is initialised with the points of the original line and the smoothing is defined solely by the internal energy. An external energy is only applied if the active contour should also maintain a distance from certain points, as in the situation when the active contour overlaps with another line in the map. Then proximity of the control points on the second line acts as the external energy pushing the active contour away from the second line.

The key difference between using active contours for crisping the field representation and the previously listed problems, is that the crisping problem lacks a clearly defined border to which the active contour could be attracted. To counter this a third energy has been introduced, which forces the active contour to contract towards the centre of the spatial relation field. The active contour now consists of three energies (equation 2). $E_{int}(s)$ maintains the active contour's shape, $E_{relation}(s)$ is the external energy defined by the spatial relation field and $E_{contract}(s)$ specifies the contraction energy.

$$E(s) = \alpha \cdot E_{int}(s) + \beta \cdot E_{relation}(s) + \gamma \cdot E_{contract}(s)$$
(2)

A greedy algorithm has been designed, that iteratively finds a solution to the crisping problem. For computational reasons, instead of working directly on the scalar values, a vector representation has been chosen. On each iteration $E_{int}(s)$ is calculated for each control point by first determining the vector from the predecessor control point to the successor control point. The mid-point of this vector is then calculated and the internal energy is defined as the vector



Figure 2: Four states in the active contour process for "north of <toponym>": Initial position, intermediate shape 1 and 2, final result (\times marks the ground toponym location)

from the current control point to that mid-point. This maintains a smooth curvature and an even spread between the control points.

 $E_{relation}(s)$ is calculated from the original field data by applying the gradient operator to it, creating a vector field, the gradient flow field. For each point in the original scalar field the gradient operator finds the neighbouring point with the largest difference in the scalar value. This defines the direction of the vector and the length is then calculated from the difference in the scalar values. $E_{contract}(s)$ is defined directly as a vector field in which all vectors point to the area within the relation field with the highest membership value and are of equal length, providing a constant contraction energy.

Each of these energies has a weight associated with it, the manipulation of which modifies the final contour form. The α weight modifies the internal energy of the active contour. Increasing the value creates a stiffer active contour, while lowering it allows sharp corners to appear. The contraction weight γ controls how far the active contour contracts. A high value leads to a smaller final result, while a smaller value produces a larger active contour. The weight β on the relation energy acts as a balance between the two other weights. For example increasing the internal energy weight α will also lead to a slightly stronger contraction. In order to maintain the same level of contraction, β is increased, maintaining the amount of contraction, but now with a more rigid active contour. The results shown in figure 2 used weights of $\alpha = 70$, $\beta = 255$, $\gamma = 110$.

After weighting each energy, the vectors are combined and the control point is then moved one cell in the direction of the total energy vector. The length of the total energy vector is not taken into account. As the control point is immediately updated, it changes its influence on the internal energy of the next control point. This means that the algorithm will find locally optimal solutions for each control point, but the results will not necessarily be globally optimal. Determining when to terminate the algorithm is a hard problem. Multiple termination criteria are under consideration, but currently a hard limit of 400 iterations is implemented.

When applying the algorithm to an image caption, after the algorithm terminates, the co-ordinates for the crisp boundary are calculated from the active contour control points based on a mapping of the ground location in the field to the actual location of the toponym from the image caption.

4.1. Evaluation

As the crisping algorithm is essentially arbitrary, it is necessary to provide a confidence value for it. This confidence value is not a measure of whether the generated shape is correct or not, but describes how confident the algorithm is that the resulting polygon is acceptable to a majority of people. The confidence function $C(s, t) \rightarrow [0, 1]$ is defined on the active contour s and a test set of valid uses of the spatial relation r. These valid uses can either be automatically calculated from an existing data set such as the Geograph data, or elicited directly from users using other methods.

$$C(s,t) = 1 - \left|\frac{\frac{count(t)}{2} - inside(s,t)}{\frac{count(t)}{2}}\right|$$
(3)

In the confidence function the confidence is highest at the point where half the points in the test set are outside and half inside the active contour. The confidence decreases as the active contour covers either less or more of the test set. The rationale behind this is that the confidence in the active contour result is highest when the number of test points inside and outside the active contour are the same. In this situation the number of people who would say that the relation extends further is in balance with the number of people who would say that the relation does not extend that far, increasing the likelihood that both groups agree that the result is an acceptable approximation of what they believe to be true. The final active contour shown in the fourth image of figure 2 has a confidence value C = 0.96.

5. Conclusion and future work

This paper presented a field-based model for representing vague areas defined by natural language spatial relations. In order to interface the model with current crisp-region based models, a crisping algorithm based on active contours is described. The model and crisping algorithm are applied to the domain of spatial language in image captions and the field creation and crisping is demonstrated on data generated with image captions taken from the Geograph project. The major advantage of the field model over current broad boundary and fuzzy models is that it allows the precise modelling of vague regions, while avoiding the complexity of fuzzy representations. As few current GI systems support vague regions, the crisping algorithm makes it possible to easily integrate the field model into existing systems and methods.

Due to the nature of the data that forms the basis for this work, the results are restricted to the context of image captioning and the scale of populated places. Future work will focus on extending the model to further contexts, spatial relations and scales. This will include dealing with differing spatial reference frames. The focus will also be on acquiring spatial relation extents directly from people and on how different languages influence these extents.

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